

# Spin foams, causal links and geometry-induced interactions

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Current theories of particle physics, including the standard model, are dominated by the paradigm that nature is basically translation invariant. Deviations from translation invariance are described by the action of forces. General relativity is based on a different paradigm: There is no translation invariance in general. Interaction is a consequence of the geometry of space-time, formed by the presence of matter, rather than of forces.

In recent years the formation of space-time on a quantum mechanical level, has been intensively studied within the framework of spin foams, following an old idea from R. Penrose. In this connection it would be appropriate to reconsider the meaning of those paradigms and attempt to apply the paradigm of general relativity to particle physics.

A spin foam model with underlying  $SO(3,2)$  symmetry is well-suited for this purpose. It represents a purely geometric model in the sense of the second paradigm. By applying perturbative methods, starting from a translation invariant first approximation, this model is reformulated in the sense of the first paradigm. It will be shown that the model then defines a space-time manifold equipped with a particle theory in the form of locally interacting quantized fields. This includes all four types of interaction: electromagnetic, weak, chromodynamics and gravitation together with realistic numerical values of the corresponding coupling constants.

## I. INTRODUCTION

Since Isaac Newton formulated his laws of motion, translation invariance has become and still is the dominant paradigm of our understanding of nature. Translation invariance is generally understood as a basic symmetry, that is more or less disturbed by the action of forces within a manybody system. Of course, such a system as a whole is usually assumed not to be under the influence of external forces. Thus translation invariance is kept up for the system as a whole. Today, this paradigm, together with Lorentz invariance, is the basis for the field theoretical formulation of the standard model of elementary particles.

It has been criticized that Newton's laws are a definition of the notion of force rather than a law of nature. This objection can be formulated as an alternative paradigm: *Nature is basically not translation invariant, but translation invariance is a useful starting point in developing a physical theory. In refining the theory corrective terms have to be added that are defined by the differences between real and fictitious geometry. These corrective terms are expressed by the concept of 'forces'.*

The well-known derivation of Newton's theory of gravitation from Einstein's theory of general relativity, demonstrates in an impressive way how a theory based on the second paradigm can be

converted into a formulation that is in agreement with the first paradigm. The derivation extracts, from the pure geometric concept of Riemannian space-time, gravitational forces as (an approximate) description of the same kinematical situation, viewed from a flat coordinate system, rather than from curved space-time.

With this beautiful example in mind, it seems natural to ask whether such a duality of concepts can be established also in particle physics. This article, therefore, takes up the following question: Can we find a geometry, not of space-time but of a Hilbert space with the following properties? It should allow to derive Minkowskian space-time as an approximate space-time manifold. At the same time it should deliver interaction terms, when we use the obtained approximate Minkowskian space-time to formulate kinematical relations.

In a way, string theories have been trying to answer similar questions for the last three decades. But they still have severe difficulties to relate their mathematical models to empirical particle physics, and there is no indication that their problems can be solved in the near future.

The concepts of spin networks and spin foams, respectively, are also aimed at the same question. Spin networks were introduced by R. Penrose [1] more than 30 years ago, in an attempt to describe the geometry of space-time in a purely combinatorial way. In recent years spin networks and spin foams have gained increased interest as an instrument in the formulation of hypothetical structures of space-time at Planck scales aiming at a consistent quantum geometry of space-time. (See for example J. C. Baez [2] for an introduction to spin networks and spin foam models and references given therein.)

At Planck scales, corresponding to a characteristic length of  $l_P = 1.6 \times 10^{-35}$  m, the usual continuous space-time manifold is assumed to break down and have to be replaced by a discrete structure. A continuous space-time manifold, equipped with reasonable physics at experimentally accessible scales, is then expected as the result of a proper large-scale approximation to a spin-network. However, in spite of the progress made in recent years it is still unknown how such an approximation can be formulated.

A concept of particle physics that is based on ‘events’ in space and time, connected by ‘causal links’, has been advocated by R. Haag [3], [4]. Along this line the comparatively new concept of *causal* spin networks has been developed by F. Markopoulou and L. Smolin [5], [6]. It has been used to study the formation of space-time using causal links to describe ‘histories’ within spin networks at Planck scales. This concept will be helpful also at ‘normal’ scales, for the interpretation of the results that we will obtain.

Despite the unsolved questions of spin networks at Planck scales, we will base our considerations on a spin foam concept but we will avoid the problems caused by the attempt to find an access to spin foams at Planck scales. Our approach will lead us, in a natural way, to space-time without the need to start from Planck scales.

Our strategy will be to make use of comparatively modest basic assumptions that are closer to well-established concepts of particle physics. At first sight these assumptions may appear as too narrow to permit a reasonable answer to the above question. It will turn out, however, that they yield just enough properties to formulate a ‘tight fitting’ realistic particle theory. On the other hand their simplicity and the close relationship to conventional and well-understood procedures will allow, a mathematically transparent transition, to empirical particle physics as formulated by the standard model.

## II. OVERVIEW

We will proceed according to the following programme:

Step 1. Find a suitable symmetry group, to be used within a spin network, that contains the homogeneous Lorentz group but not translations as a subgroup.

Step 2. Approximate, in a proper way, this symmetry group by the Poincaré group.

Step 3. Identify the differences between exact symmetry group and Poincaré group.

Step 4. Bring these differences into a form that can be compared with familiar descriptions, preferably with ‘interaction terms’ of the standard model.

Step 5. Compare these interaction terms with those of the standard model, if existent, and identify the types of forces that are defined thereby.

A good candidate for such a basic symmetry group is the well-known de Sitter group  $SO(3,2)$ , which has been used mainly in cosmological models throughout many decades. It does not contain translations as a subgroup but does contain the Lorentz group as a subgroup and can be approximated by the Poincaré group with help of the method of group contraction.

The following study is based on a multiparticle system, defined as a spin foam model constructed from spin-1/2 representations of the de Sitter group  $SO(3,2)$ . Let  $\mathcal{H}$  be the Hilbert space of this multiparticle system. Group contraction then yields a ‘tangential’ Minkowskian space-time manifold equipped with a multiparticle system of non-interacting massive lepton-like particles, or ‘Dirac’ particles for short. Let  $\mathcal{H}_O$  be its Hilbert space. This will realize steps 1 through 2.

According to step 3, several corrective terms of different structure will be identified. These terms will be used later to refine the approximation obtained by group contraction. These refinements will lead us, step by step, from the contraction limit back towards the exact symmetry of  $SO(3,2)$ . But in doing so, and this is crucial for our approach, we will retain the translation invariant  $\mathcal{H}_O$  as the mathematical basis of the description.

In order to compare the corrective terms with the interaction terms of the standard model, we will treat the corrections as a perturbation to the (Poincaré invariant) multiparticle system defined in  $\mathcal{H}_O$ , using conventional Fock space methods.

Making these corrective terms suitable for a perturbation treatment will require their linearization by the introduction of auxiliary quantized fields that act as relativistic potentials.

This will bring one of these corrections into a form, identical to the interaction term of the familiar perturbative formulation of quantum electrodynamics (QED).

In contrast to the standard model, the  $SO(3,2)$  model does not contain free parameters that can be adjusted to the experimental values of coupling constants. Therefore, the coupling constants are uniquely determined by the model. Their evaluation should either support or fault the suspected relation to known interactions. For this purpose an estimate for the coupling constant for the QED-like interaction term based on the  $SO(3,2)$  model is derived. The estimate reproduces Wyler's heuristic formula [7], which is known to deliver the value of the fine structure constant with a high degree of precision.

The second corrective term cannot be evaluated in the same way, simply because the standard model does not contain this type of interaction. This interaction is characterized by causing a curving of the space-time manifold, obtained before by group contraction. This curving is proportional to the distribution of matter, but does not depend on internal quantum numbers. Based on covariance arguments, it can be concluded that a classical correspondence limit to this interaction, must result in field equations of the type of general relativity. So we have good reasons to regard this interaction as a form of quantum gravity.

These two contributions do not cover all corrections that have to be applied to follow the way back to the exact  $SO(3,2)$  symmetry. We will find which correction defines weak interaction and what the model can tell us about the nature of neutrinos. Finally, we will identify quark-like states that give rise to an interaction of the type of quantum chromodynamics.

To avoid misunderstandings, we will *not* use the de Sitter group to describe a symmetry of space-time similar to the cosmological model of de Sitter space. Instead we will use the de Sitter group as

a symmetry group of a Hilbert space without any regard to possible consequences for a space-time structure in the large or small. Actually, we will not even assume the existence of a predetermined space-time continuum.

### III. SO(3,2) BASED SPIN FOAM MODEL

Given a Lie group  $G$ . Then a spin network is defined by the following properties (see e.g. [2]):

**Definition:** A spin network is a triple  $\Psi = (\gamma, \rho, \tau)$  consisting of:

1. a graph  $\gamma$ ,
2. for each edge of  $\gamma$ , an irreducible representation  $\rho_e$  of  $G$ ,
3. for each vertex  $v$  of  $\gamma$ , an intertwining operator

$$\tau_v : \rho_{e_1} \otimes \cdots \otimes \rho_{e_n} \rightarrow \rho_{e'_1} \otimes \cdots \otimes \rho_{e'_n}, \quad (1)$$

where  $e_1, \dots, e_n$  are edges incoming to  $v$  and  $e'_1, \dots, e'_n$  are the edges outgoing from  $v$ .

Notice that this is the definition of an abstract spin network, not embedded in any space-time manifold.

Spin networks are general frameworks that have to be filled with contents. We intend to formulate a model, where  $G = SO(3,2)$  and identical irreducible spin-1/2 representations of the  $SO(3,2)$  are attached to each edge.

Let  $l_{ab}$ ,  $a, b = 0, 1, 2, 3, 4$ , be representations of the infinitesimal generators of  $SO(3,2)$  by operators in a quantum mechanical state space and let  $l_{\mu 4}$  be those operators that in the contraction limit converge towards the momentum operators  $p_\mu$  of the Poincaré group. Then  $l_{\mu\nu}$  are the operators of the Lorentz subgroup with  $\mu, \nu = 0, 1, 2, 3$  with the following commutation relations

$$[l_{\mu\nu}, l_{\rho\sigma}] = -i[g_{\mu\rho}l_{\nu\sigma} - g_{\mu\sigma}l_{\nu\rho} + g_{\nu\sigma}l_{\mu\rho} - g_{\nu\rho}l_{\mu\sigma}]. \quad (2)$$

with  $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ . The operators  $l_{\mu 4}$  satisfy these commutation relations

$$[l_{\mu 4}, l_{\nu 4}] = -ig_{44}l_{\mu\nu} \text{ with } g_{44} = +1 \quad (3)$$

and

$$[l_{\mu\nu}, l_{\rho 4}] = i[g_{\nu\rho}l_{\mu 4} - g_{\mu\rho}l_{\nu 4}]. \quad (4)$$

To represent spin we take advantage of the fact that the 4x4-matrices  $s_{\mu\nu}$  and  $s_{\mu 4}$ , built from Dirac matrices,

$$s_{\mu\nu} := \frac{1}{2}\sigma_{\mu\nu} \text{ and } s_{\mu 4} := \frac{1}{2}\gamma_\mu \quad (5)$$

with

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \quad (6)$$

and

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (7)$$

satisfy the commutation relations of the  $SO(3,2)$ .

We also will need

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3, \quad (8)$$

which anticommutes with all  $\gamma_\mu$ . In the Weyl representation  $\gamma_5$  is diagonal

$$\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \quad (9)$$

A Spin-1/2 representation is obtained from the direct product of an infinite-dimensional ‘orbital’ part and a finite-dimensional ‘spin’-part. The operators of this representation are given by

$$j_{ab} := l_{ab} + s_{ab}. \quad (10)$$

The motivation for using such a representation is that in the (massive) contraction limit (see next section) it reduces to a well-understood representation of the Poincaré group. This describes Dirac particles with a given mass  $m$  in Minkowski space-time. This will give the model a chance to be realistic in this respect. For the moment we can assume that at all vertices  $n = n' = 2$ . At each vertex two particles are allowed to exchange their orbital and, possibly, spin quantum numbers with an amplitude that will have to be determined. By the end of this article we will have collected enough information to come to a precise definition of the vertices.

When we perform the contraction limit, the model will exhibit three space-like and one time-like dimension. Therefore, on a time cut the  $SO(3,2)$  spin network defines a ‘quantum 3-geometry’. The inclusion of the time-like dimension extends the spin network to a spin foam with a ‘quantum 4-geometry’. The vertices of the spin foam will be regarded as ‘events’ in space and time, which refer to a sharp position in a space-time manifold still to be defined.

The author considers the formulation of this  $SO(3,2)$  model as an intermediate step on the way to a model that is finally based on *finite*-dimensional spin representations of  $SO(3,2)$ . In this article, however, no attempt has been made to extend the model in this direction.

Instead of using the notions of spin network and spin foams, we could simply talk about multi-particle systems, since we intend to use the former notions in the sense of the latter. The author has decided to use the ‘modern’ notions, firstly because these are commonly connected to what this article aims at, namely to derive space-time and interactions from a purely geometric concept. Secondly, because the article will clarify some aspects of spin foams.

#### IV. GROUP CONTRACTION

The method of group contraction was mathematically formulated by E. Inönü and E. P. Wigner [8] in 1953. By group contraction the Poincaré group is obtained as “in some sense, a limiting case” of the de Sitter group if the “de Sitter radius”  $R$  approaches infinity.

When these authors formulated the contraction limit, they had in mind the cosmological model of de Sitter space-time. Our interest is merely to obtain a suitable Poincaré invariant mathematical basis for a following perturbative expansion.

Group contraction is defined as a restriction of the operators to a domain of the Hilbert space, where the expectation values of  $l_{\mu 4}$  are large compared to those of  $l_{\mu\nu}$ , so that for absolute values of amplitudes between states  $\phi$  and  $\phi'$  of this domain the following relation holds

$$|\langle\phi|l_{\mu 4}|\phi'\rangle| \gg |\langle\phi|l_{\nu\rho}|\phi'\rangle| \text{ for all } \mu, \nu, \rho. \quad (11)$$

As a consequence of the commutation relation (3), the operators  $l_{\mu 4}$  can then be approximated by commuting operators  $p_\mu$  that are identified as the translation operators of the Poincaré group  $P(3, 1)$ .

E. Inönü and E. P. Wigner [8], see also F. Gürsey [9], have formalized relation (11) by rescaling the operators  $l_{\mu 4}$  by a factor  $1/R$

$$\Pi_\mu = \frac{1}{R} l_{\mu 4} \quad (12)$$

and defining Poincaré momentum by the limit

$$p_\mu = \lim_{R \rightarrow \infty} \Pi_\mu. \quad (13)$$

With these definitions the commutation relations of  $l_{\mu 4}$  are

$$[\Pi_\mu, \Pi_\nu] = \frac{-i}{R^2} l_{\mu\nu} \quad (14)$$

and, therefore,

$$[p_\mu, p_\nu] = 0. \quad (15)$$

F. Gürsey [9] has formulated the analogue of eq. (12) for spin-1/2 representations

$$\Pi_\mu = \frac{1}{R} l_{\mu 4} + \frac{1}{2R} \gamma_\mu. \quad (16)$$

(See also C. Frønsdal et al [10].) The  $\gamma$ -term was named “momentum spin” by F. Gürsey.

For  $R \rightarrow \infty$  it is assumed that the operators  $l_{\mu 4}$  grows proportionally to  $R$  so that the  $l_{\mu 4}$  term in (16) remains finite and converges to  $p_\mu$ . The  $\gamma_\mu$  term then becomes a second order and can be neglected if  $R \rightarrow \infty$ .

By performing the contraction limit we have obtained the algebra of the Poincaré group as a kind of high energy approximation to the algebra of  $SO(3,2)$ . In this approximation the multiparticle system, on which we base our analysis, is replaced by a system of Dirac particles with Hilbert space  $\mathcal{H}_\mathcal{O}$ .

The scaling factor  $R$  serves a twofold purpose. It ensures that relation (11) is satisfied and it gives the momentum operator the dimension of an inverse length, if  $R$  is understood as the ‘radius’ of de Sitter space-time.

Since our model is not related to de Sitter space-time we will not make further use of  $R$ , but replace this factor by 1. This means that we will treat the operator  $p_\mu$  like  $l_{\mu\nu}$  as dimensionless. Its eigenvalues are pure numbers. In the following, instead of using a scaling factor, we will make use of relation (11), which serves the same purpose. More about dimensions in the following section.

After dropping the factor  $1/R$  we can rewrite (16) in the form

$$\Pi_\mu = l_{\mu 4} + \frac{1}{2} \gamma_\mu. \quad (17)$$

It is well-known that from momentum eigenstates  $|\mathbf{p}\rangle$ ,  $\mathbf{p} = (p_1, p_2, p_3)$ , localized states  $|\mathbf{x}, t\rangle$  can be constructed that are eigenstates of a ‘position’ operator  $X_k, k = 1, \dots, 3$ ,

$$|\mathbf{x}, t\rangle := (2\pi)^{-3/2} \int d^3p e^{ipx} |\mathbf{p}\rangle \quad (18)$$

with

$$X_k |\mathbf{x}, t\rangle = x_k |\mathbf{x}, t\rangle. \quad (19)$$

The parameter space of  $\mathbf{x}, t$  has then the structure of 4-dimensional Minkowskian space-time. Thus, formally, space-time in our model is generated in a very unspectacular way, as an approximate extract



of the geometrical structure of the exact Hilbert space  $\mathcal{H}$ . Below, we will give this formal definition a precise physical meaning.

## V. DIMENSION OR NOT DIMENSION

Physical quantities like length or momentum usually have a ‘dimension’, which means that their measurement is based on an instruction how to compare the quantity to be measured with a given standard. The result of the measurement is still a pure number, but information is added to this number by which means it was obtained.

It may, therefore, cause some confusion that we have introduced momentum as a dimensionless quantity.

What does this mean? Actually, we intend to do nothing other than described before. To determine the momentum of a particle in tangential space-time, we can count the nodes of its wave function within a given area of space-time. The number of nodes is then proportional to the momentum quantum numbers. Counting nodes requires an agreement over which area or length, respectively, we have to perform the counting. This gives the momentum a ‘dimension’ in the same sense as stated before.

The other way round, we can also measure a length in tangential space-time, by comparing it with an agreed number of nodes of a given momentum standard. The reader will realize that this instruction is used in the modern definition of the meter by a certain number of krypton wavelengths.

So much for how dimensions can be attached to ‘dimensionless’ quantities.

## VI. CORRECTIVE TERMS

We now start to reconstruct the exact algebra of  $SO(3,2)$  from the algebra of the Poincaré group. Throughout this paper we assume that relation (11) is valid. In the first step we add the term  $\frac{1}{2}\gamma_\mu$  to the momentum operator. We then obtain a corrected operator in the form

$$t_\mu := p_\mu + \frac{1}{2}\gamma_\mu. \quad (20)$$

If we add another correction given by

$$c_\mu := l_{\mu 4} - p_\mu, \quad (21)$$

we will have fully reconstructed the original operator  $j_{\mu 4}$  of  $SO(3,2)$ . Since both corrective terms have a significantly different structure it makes sense to analyze their contributions separately. So

let us first try to understand the implications of the spin part and come back to the term  $c_\mu$  later.

Since in (20) the spin part is small compared to the orbital part and a well-known theory is available for the orbital part, a perturbational approach is indicated, which treats the spin part as a small perturbation to a conventional Dirac theory. In the following we will develop such a perturbational access to the physical implications of the operator (20).

## VII. CONSTRAINTS IN MULTIPARTICLE SYSTEMS

Consider the operator  $j_{ab}j^{ab}$ ,  $a, b = 0, 1, 2, 3, 4$  (summation over  $a, b$ ), where  $j_{ab}$  are given by (10). This is one of the invariant operators of the  $SO(3,2)$  algebra, which means that it commutes with all  $j_{ab}$ .

In a multiparticle system we deal with operators

$$J_{ab} := j_{ab} + j'_{ab} + j''_{ab} + j'''_{ab} + \dots \quad (22)$$

that are defined as the sum of particle-individual operators. If this system is isolated, which means that it can be described by an irreducible representation of  $SO(3,2)$ , then the invariant operator  $J_{ab}J^{ab}$  can be assigned a fixed c-number.

For our spin foam, or any isolated subsystem of it, the evaluation of the relation

$$J_{ab}J^{ab} = \text{const} \quad (23)$$

can, therefore, lead to ‘selection rules’ for possible ‘transitions’ at its vertices.

In the following we will approximately evaluate the multiparticle operator relation (23) by using (20) as an approximation to  $j_{\mu 4}$  and taking (11) into account, collecting terms of the magnitude  $p^2$  and  $p$  and ignoring terms of lower magnitude. We then obtain the expression (written out for two particles with momentum  $p$  and  $p'$ )

$$\begin{aligned} & p_\mu p^\mu + 2p_\mu p'^\mu + p'_\mu p'^\mu \\ & + \frac{1}{2}(\gamma_\mu p^\mu + \gamma_\mu p'^\mu + \gamma'_\mu p^\mu + \gamma'_\mu p'^\mu) \\ & + \dots = \text{const}, \end{aligned} \quad (24)$$

which can be written in the form

$$T_\mu T^\mu = \text{const.} \quad (25)$$

with

$$T_\mu := t_\mu + t'_\mu + \dots, \quad (26)$$

where  $t_\mu, t'_\mu, \dots$  are the particle-individual operators given by (20).

In other words, (25) defines a constant-of-motion. Constants-of-motion have always been useful means to study the internal kinematics of a physical system.

In the Hilbert space  $\mathcal{H}_\mathcal{O}$ , which will be used below for a perturbational treatment of (25), there is another constant operator

$$P_\mu P^\mu = \text{const.}, \quad \text{where } P_\mu := p_\mu + p'_\mu + \dots. \quad (27)$$

In  $\mathcal{H}_\mathcal{O}$   $P_\mu$  commutes with  $T_\mu$ . So the modulus  $P^\mu P_\mu$  is also a constant with respect to any transformation that is generated by  $P_\mu, T_\mu$  or any generator of the homogeneous Lorentz subgroup.

The fact that both  $P^2$  and  $T^2$  are constants-of-motion does not mean that both represent invariant operators in the sense of representation theory. This is rather a consequence of the perturbation algorithm: The c-number value of  $P^2$  defines the Hilbert space  $\mathcal{H}_\mathcal{O}$  that is used to evaluate  $T^2$ . It is a c-number by construction. The c-number value of  $T^2$  really defines a constraint.

The constancy of  $P^2$  enables us, to separate (24) into contributions that are quadratic in  $p_\mu$  and those that are linear. The latter, therefore, form another constant expression

$$\begin{aligned} & \gamma_\mu(p^\mu + p'^\mu + \dots) \\ & + \gamma'_\mu(p'^\mu + p^\mu + \dots) \\ & + \dots \\ & = \text{const.} \end{aligned} \quad (28)$$

Here we find terms that represent the operators of a Dirac equation of individual particles, and other terms - like  $\gamma_\mu p'^\mu$  - that provide for a connection between pairs of particles. Whereas the former belong to a Poincaré covariant description of a multiparticle system in Minkowski space-time, the latter can be understood as a perturbation to the Poincaré covariant system. So we rearrange the terms

$$\begin{aligned} & \gamma_\mu(p^\mu + a^\mu) \\ & + \text{similar terms for the other particles} \\ & = \text{const.} \end{aligned} \quad (29)$$

with the perturbation term

$$a^\mu = \sum p'^\mu \text{ (sum over all particles except the first).} \quad (30)$$

$a^\mu$  describes one of the differences between Poincaré and exact symmetry. With this we have realized step 3 of our programme for the first corrective term. Next we will bring this result into a form that can be compared with familiar formulations of multiparticle physics.

### VIII. STRUCTURE OF TWO-PARTICLE STATES

Since the perturbation term  $\gamma_\mu a^\mu$  is basically a sum over two-particle operators, we will have to make use of two-particle states. So let us spend a short look at their general structure (ignoring spin variables).

Let

$$|\mathbf{P}\rangle = \int \frac{d^3p}{p_0} \frac{d^3p'}{p'_0} C(\mathbf{p}, \mathbf{p}') |\mathbf{p}, \mathbf{p}'\rangle \quad (31)$$

be a two-particle state with 4-momentum  $P = (\mathbf{P}, P_0)$  of a two-particle representation of  $P(3,1)$  with  $P^2 = M^2$  in a state space  $\mathcal{H}_M$ . The two-particle states  $|\mathbf{p}, \mathbf{p}'\rangle$  belong to the direct product of one-particle states  $|\mathbf{p}\rangle$  with 4-momentum  $p = (\mathbf{p}, p_0)$ .

With  $p = k - q$ ,  $p' = k + q$ ,  $2k = P$  we can rewrite

$$|\mathbf{P}\rangle = \int \frac{d^3q}{q_0} \tilde{C}(\mathbf{q}) |\mathbf{k} - \mathbf{q}, \mathbf{k} + \mathbf{q}\rangle. \quad (32)$$

From

$$p^2 = p'^2 = m^2, \quad (33)$$

where  $m$  is the particle mass, and

$$P^2 = (p + p')^2 = M^2 \quad (34)$$

we obtain

$$q^2 = m^2 - \frac{1}{4}M^2, \quad (35)$$

$$q_0^2 = m^2 - \frac{1}{4}M^2 + \mathbf{q}^2 \quad (36)$$

and

$$kq = 0. \quad (37)$$

Conditions (35) through (37) express the fact that  $|\mathbf{P}\rangle$  is a state of an two-particle representation characterized by the total mass  $M$ .

We can formulate the state (31) also in terms of wave functions

$$e^{-iPx} = \int \frac{d^3q}{q_0} \tilde{C}(\mathbf{q}) e^{-i(k-q)x} e^{-i(k+q)x}, \quad (38)$$

which are obtained by formally multiplying the ket-states  $|\mathbf{P}\rangle$ ,  $|\mathbf{k} - \mathbf{q}\rangle$  and  $|\mathbf{k} + \mathbf{q}\rangle$  by its associated bra-states  $\langle x|$ .

The momenta  $p$  and  $p'$  of each term under the integral in (31) adds up to  $P$ . In other words, the individual momenta are ‘entangled’ within two-particle states.

## IX. INTERACTION TERM IN FOCK SPACE

Let us now return to our multiparticle Hilbert space  $\mathcal{H}_\mathcal{O}$ , which has been defined by the direct product of one-particle state spaces, which in turn are defined by momentum eigenstates that satisfy the Dirac equation. We reformulate  $\mathcal{H}_\mathcal{O}$  with the help of standard Fock space methods.

The ‘free’ part of our system is easily converted into a Fock space formulation following the usual ‘second quantization’ of the Dirac field. We will skip this step and refer to standard textbooks (see e.g. [11]). The field operator of the Dirac field (taken from this reference) has the form

$$\psi(x) = (2\pi)^{-3/2} \int d^3p \left( b_s(\mathbf{p}) u_s(\mathbf{p}) e^{-ipx} + d_s(\mathbf{p})^\dagger v_s(\mathbf{p}) e^{ipx} \right). \quad (39)$$

A similar expression defines the Dirac adjoint operator  $\bar{\psi}(x)$ .  $b_s^\dagger(\mathbf{p}), b_s(\mathbf{p})$  are electron emission and absorption operators,  $d_s^\dagger(\mathbf{p}), d_s(\mathbf{p})$  are the corresponding operators for positrons. They satisfy the usual anticommutation relations of the Dirac field.

As a first attempt we represent our two-particle perturbation terms  $\gamma_\mu p'^\mu$  in Fock space in the following form

$$\int d^3x d^3x' \bar{\psi}(x) \gamma_\mu \psi(x) \bar{\psi}(x') p'^\mu \psi(x'). \quad (40)$$

This Fock space operator is not yet adapted to its immediate insertion into a perturbation calculation since it has two major flaws.

Firstly: It is built as a product of two Fock operators. This will result in a non-linear equation for  $\psi(x)$ . Such an equation cannot be treated by a standard perturbation algorithm. We will find a solution to this problem by a linearization method.

Secondly: Remember that a standard quantum mechanical perturbation calculation takes place within the given Hilbert space  $\mathcal{H}_\mathcal{O}$ . This requires the combination of a perturbation term with a

projection operator into  $\mathcal{H}_O$  (see e.g. [13]). Therefore, we have to incorporate a suitable projection mechanism. Since the interaction term is a two-particle operator we can base this mechanism on two-particle subspaces of  $\mathcal{H}_O$ . As shown below, we can implement a suitable projection mechanism by collecting only those terms of (40) that contribute if we evaluate this operator for two-particle states with total momentum  $P$ ,  $P^2 = M^2$ ,  $M$ =c-number, of a given (irreducible) two-particle state space  $\mathcal{H}_M$ . This will ensure the correct projection onto  $\mathcal{H}_O$  also in the general case of more than two particles. One can convince oneself that this procedure is also necessary to ensure such a projection in the general case.

## X. IMPLEMENTING A PROJECTION ONTO THE HILBERT SPACE

Consider the following contribution to (40)

$$\dots \bar{b}(\mathbf{p} + \mathbf{k}) \gamma_\mu b(\mathbf{p}) \bar{b}(\mathbf{p}' - \mathbf{k}') p^\mu b(\mathbf{p}') \dots \quad (41)$$

(We have omitted the factors of  $u_s$  and  $v_s$  for a moment.) If we evaluate this operator for a two-particle state, then only terms with  $\mathbf{k} = \mathbf{k}'$  will be involved, as a consequence of momentum entanglement within two-particle states.

This is true for every state of  $\mathcal{H}_M$  with a given momentum  $P$ . And since every two-particle state of  $\mathcal{H}_M$  can be represented by a superposition of two-particle momentum eigenstates, it is generally valid. So we can drop the restriction to a fixed  $P$  and collect all contributions that belong to the same  $\mathbf{p}$  and  $\mathbf{k}$ . Hence, we can write

$$\dots \bar{b}(\mathbf{p} + \mathbf{k}) \gamma_\mu b(\mathbf{p}) a^\mu(\mathbf{k}) \dots \quad (42)$$

with

$$e a^\mu(\mathbf{k}) := \int dV(p') \bar{b}(\mathbf{p}' - \mathbf{k}) p^\mu b(\mathbf{p}'), \quad (43)$$

where  $dV(p')$  indicates a summation over all terms that contribute to a given  $\mathbf{k}$ . As a precaution we have included a normalization factor  $e$  into the definition of  $a^\mu$  that will have to be determined after we have decided about the normalization of  $a^\mu$ . An analogous consideration is valid for positron and mixed terms.

## XI. SPLITTING THE INTERACTION TERM

Let us analyze the meaning of  $a^\mu(\mathbf{k})$  in more detail. If we evaluate (42) for a two-particle state, the second particle will contribute a complex amplitude given by the expectation value of  $a^\mu(\mathbf{k})$

that acts as a multiplicative weight to the expectation value of the first particle term. This weight depends on  $\mathbf{k}$  and fully describes the contribution of a second particle to the total expectation value. As long as our focus is on the first particle, then all we need to know about other particles are the complex weights that apply to the expectation values of particle one. To keep track of the weighting factors that apply to each ‘transition’  $p \rightarrow p + k$  in (42) we need a suitable ‘bookkeeping’ system.

We can establish such a bookkeeping system by introducing an auxiliary Fock space with operators that emit and absorb quanta with momentum  $\mathbf{k}$ . If we prepare a state in this Fock space by applying an emission operator multiplied by a complex amplitude onto the vacuum state, then a later application of an absorption operator will redeliver this amplitude. This is exactly what we need.

We have some freedom in doing this, as long as the system is able to keep track of the amplitudes of the momenta  $\mathbf{k}$ . So let us replace the operator  $a^\mu(\mathbf{k})$  of (43) by an operator  $A^\mu(\mathbf{k})$  of our bookkeeping system. This replacement means that now  $A^\mu(\mathbf{k})$  does not act on a fermions Fock state but rather on an ‘intermediate’ state in the bookkeeping Fock space. By this trick we have split the interaction term into two parts: The first term acts on the first particle and places a momentum  $\mathbf{k}$  into the bookkeeping registry, the second term takes the same momentum from the registry and acts onto the second particle, and vice versa. Notice that each half of the interaction term now has the form of a  $\gamma_\mu$  vertex, but from the view of each particle the other one seems to have a  $p^\mu$  vertex in agreement with the form (41) of the original interaction term.

In this way we have linearized the ‘equation of motion’ for  $\psi(x)$  at the expense of introducing another quantized field.

In their function as Fock space absorption and emission operators  $A_\mu$  and its adjoint  $A_\mu^\dagger$  have to satisfy the following commutation relations

$$[A^\mu(\mathbf{k}), A^\nu(\mathbf{k}')^\dagger] = \delta^{\mu\nu} \delta(\mathbf{k} - \mathbf{k}'). \quad (44)$$

Then  $A^{\mu\dagger}(\mathbf{k})$  are emission operators and  $A^\mu(\mathbf{k})$  absorption operators for quanta with momentum  $\mathbf{k}$ .

We define the following operators known from the conventional formulation of quantum electrodynamics (see e.g. [11])

$$A^j(x) = (2\pi)^{-3/2} \int \frac{d^3k}{k^0 \sqrt{2}} \left( A^j(\mathbf{k}) e^{-ikx} + A^j(\mathbf{k})^\dagger e^{ikx} \right), \quad j = 1, 2, 3, \quad (45)$$

and

$$A^0(x) = (2\pi)^{-3/2} \int \frac{d^3k}{k^0 \sqrt{2}} i \left( A^0(\mathbf{k}) e^{-ikx} + A^0(\mathbf{k})^\dagger e^{ikx} \right). \quad (46)$$

$k^0$  shall be determined by condition (36) when these operators are evaluated within two-particle states. (In the ‘free radiation field’  $k^0$  is ‘on-shell’:  $k^0 = |\mathbf{k}|$ .)

Coming back to the expression (42), we add space-time dependencies to the emission and absorption operators, as prescribed by (38). By making use of  $A^\mu$  now we obtain

$$\dots \bar{b}(\mathbf{p} + \mathbf{k}) e^{i(p+k)x} \gamma_\mu b(\mathbf{p}) e^{-ipx} A^\mu(\mathbf{k}) e^{-ikx} \dots \quad (47)$$

Notice that the correct space-time dependency of  $A^\mu$  is determined by (43).

After inserting the spin functions  $u_s(\mathbf{p})$  and  $v_s(\mathbf{p})$  these terms and the corresponding positron and mixed terms add up to a Fock operator in the form

$$e \int d^3x : \bar{\psi}(x) \gamma_\mu \psi(x) : A^\mu(x), \quad (48)$$

where  $::$  stand for normal ordering of emission and absorption operators (all emission operators stand left of all absorption operators). This is the form of the interaction term of quantum electrodynamics (QED).

Actually, we have done something very familiar from classical and quantum mechanics: We have linearized a two-body problem by introducing a potential that describes the action of particle 2 on particle 1 and vice versa. This potential has not been obtained by a formal ‘quantization rule’ applied to a classical potential, but by explicit construction on the quantum mechanical level. This provides us with a full insight into its mathematical and physical implications.

The interaction term (48) uniquely defines the structure of QED. Therefore, the full machinery of QED is available to analyze the effect of this term in a perturbation calculation. The result of this analysis is well-known and signifies that a Dirac particle within the SO(3,2) model shows the properties of an electrically charged particle.

## XII. ITERATED FIELDS AND FEYNMAN GRAPHS

The explicit construction of the interaction term, makes it possible to give a well founded interpretation of the mathematical and physical contents of the iterated field operator that we obtain from perturbation calculations.

Since the interaction term is identical to that of QED, the application of the perturbation algorithm will produce all those terms that are known from QED and that are represented there by



Feynman graphs. Some of these graphs or components of graphs, respectively, have been given the interpretation of pair creation, vacuum fluctuation and, not to forget, of ‘free’ photons.

Let us start with the notion of photons. The photon field in the  $SO(3,2)$  model has been introduced as a means to describe the exchange of momentum between two fermions. A photon connects *two* vertices in the sense of a causal link. Therefore, the existence of a ‘free’ photon is only due to the fact that it has been emitted at a point  $x$  in space-time *and* that it will be absorbed at another point  $x'$  - possibly far away from  $x$ . As we know from basic nuclear physics the exchange of massive particles leads to short distance interactions. (This information can also be obtained from the structure of the propagators representing internal photon lines.) Therefore, it is understandable that only mass zero photons can be exchanged over a large distances. This means: ‘free’ photons are ‘on-shell’ quanta of the vector potential with mass zero.

This interpretation clearly shows that the electromagnetic interaction basically is a long range interaction, with photon representing *causal links* rather than ‘real’ particles. Relativistic causality in this connection is guaranteed by the causal properties of the commutation functions.

Nevertheless, the introduction of the photon field allows us to describe the interaction by a local (point like) electron-photon vertex. But keep in mind that the basic interaction process always consists of two such vertices.

One word about external fermion lines corresponding to ‘asymptotic states’: These states in our model simply mean one-particle states in  $\mathcal{H}_O$ . There is no need to complicate matters in defining asymptotic states as a limit for  $t \rightarrow \pm\infty$ .

Now we come to ‘pair creation’. This process is characterized by a Feynman graph where, as a result of a scattering process, there are two outgoing fermion lines - an electron line and a positron line. Since the Fock operator that represents the interaction term, by construction, cannot physically create or annihilate fermions, we obviously have to accept the fact that, as a consequence of relativistic covariance, there is the possibility of scattering not only in space-like but also in time-like directions. Scattering in space-like directions means (in the centre-of-mass system) a change of sign of a space-like component of a 4-momentum. Scattering in time-like direction correspondingly means a change of sign of the time-like component of a 4-momentum. As in the space-like case, after the scattering event particles change their direction of movement, in the time-like case an electron after the scattering event moves backwards in time and is called a positron from now on.

The picture of a particle of negative energy running backwards in time is Feynman’s view of a positron [12]. Here it develops in a most natural way. This ‘running backwards in time’ is the deeper

reason why, for states of negative energy the absorption operator in (39) had to be replaced by an emission operator.

Now to ‘vacuum fluctuation’. Some Feynman graphs contain closed fermion loops that have been interpreted as creation and annihilation of virtual electron-positron pairs from the ‘vacuum’. ‘Virtual pairs’ have long been considered as a clear indication that a theory of elementary particles cannot be established without the introduction of relativistic quantized fields, and that such fields theories automatically mean a theory with an infinite number of particles.

What does the  $SO(3,2)$  model tell us about ‘vacuum fluctuation’?

Closed loops are a consequence of the perturbative iteration of a Fock operator which, by construction, cannot create fermions from the vacuum. If this Fock operator is iterated it still cannot create fermions. Again, if this Fock operator is applied to two-particle states, there are exactly two particles involved and not more. A down-to-earth interpretation of such loops is easily obtained if we remember that internal lines in a Feynman graph are generated by interchanges of emission and absorption operators, by using their commutation relations. These interchanges essentially deliver delta-functions for the momenta, which, within the perturbation algorithm, are extended in a covariant way to 4-momentum space. Therefore, internal lines simply keep track of 4-momentum within the perturbation algorithm and have nothing to do with particles, not even with ‘virtual’ ones. This is in contrast to external lines which do represent one-particle and one-photon states.

In the past the highly imaginative picture of a vacuum swirling with virtual pairs, has proven more attractive than a rational analysis of the perturbation algorithm. Unfortunately, this picture has influenced our way of looking at particle physics throughout five decades.

### XIII. GAUGE INVARIANCE

In the standard formulation of QED an interaction with a ‘gauge field’ is introduced by postulating gauge invariance of second kind. In contrast to this rather formal procedure our evaluation of the  $SO(3,2)$  model lead us to an explicit construction of the interaction term and the ‘bookkeeping field’ from known elements of the electrons Fock space. This enables us to *prove* gauge invariance of the second kind rather than only postulate it.

Consider the two-particle wave function (38) and multiply the first one-particle wave function on the right side by  $e^{i\Lambda(x)}$  and the second by  $e^{-i\Lambda(x)}$ . Obviously, this leaves the two-particle wave function invariant. Now apply the first term in (29) with all momentum operators expressed by  $-i\partial^\mu$  to this ‘gauge transformed’ two-particle state. Then  $-i\partial^\mu$  applied to the wave function of the first

particles contributes a term  $\partial^\mu \Lambda(x)$  whereas  $a^\mu$  applied to the wave function of the second particles by making use of (30) delivers the same term with opposite sign. Both terms cancel each other. This is exactly what is meant by gauge invariance of the second kind.

#### XIV. ESTIMATE OF THE COUPLING CONSTANT

Unlike the standard formulation of QED, where the coupling constant enters as a free parameter that has to be determined by the experiment, our approach does not leave room for any free parameter. This means that the coupling constant  $e$  is determined by the theory and, therefore, should be calculable.

The coupling constant is defined by the normalization factor  $e$  in (43). After replacing  $a^\mu(\mathbf{k})$  by  $A^\mu(\mathbf{k})$  the normalization of the Fock operators in (43) is fixed by their commutation relations. Then  $e$  can be determined by correctly ‘counting’ all contributions to the integral - in other words: by a careful analysis of the volume element of the integral in (43).

More than 30 years ago A. Wyler [7] discovered that the fine-structure constant  $\alpha$  can be expressed by volumes of certain symmetric spaces. Being a mathematician he was not able to put his observation into a convincing physical context. Therefore, his work was criticized as fruitless numerology [17].

Wyler’s idea was picked up later by F. D. Smith, Jr. [18] who extended Wyler’s heuristic approach into a general scheme based on a fundamental Spin(8) symmetry. Smith was then able to express coupling constants and relations of particle masses by characteristic volumes with a remarkable degree of precision.

Let us see how far our model will lead us and whether we possibly can find a physical explanation for these authors’ observations.

The following will be oriented to a scattering process (Møller scattering), which means two vertices. Therefore, the factor  $e$  and the volume element of (43) will enter twice into the estimate.

In (43) we already have parameterized the contributions to the interaction term by the parameters  $\mathbf{p}'$  and  $\mathbf{k}$ . The way in which the parameters  $\mathbf{k}$  are used in the perturbation calculation, defines the parameter space of  $\mathbf{k}$  as (a subspace of) the Euclidean  $R^3$ . If we keep  $\mathbf{k}$  fixed, we are left with the integral over  $\mathbf{p}'$  and our task will be to determine the multiplicity or the integration volume, respectively, of the contributions with respect to  $\mathbf{p}'$ .

The basis for the evaluation of the integration volume is the particle momentum and the homogeneous Lorentz group acting on the particle momentum. The  $SO(3,1)$  acts transitively on a particles

mass shell

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 = m^2. \quad (49)$$

The independent parameters  $p_1, p_2, p_3$  span a 3-dimensional parameter space. For a two-particle state of a representation with mass  $M$  we have instead

$$(p_0 + p'_0)^2 - (p_1 + p'_1)^2 - (p_2 + p'_2)^2 - (p_3 + p'_3)^2 = M^2. \quad (50)$$

We can convert this into

$$\begin{aligned} p_0^2 + p_0'^2 - p_1^2 - p_1'^2 - p_2^2 - p_2'^2 - p_3^2 - p_3'^2 \\ + 2p_0p'_0 - 2p_1p'_1 - 2p_2p'_2 - 2p_3p'_3 = M^2. \end{aligned} \quad (51)$$

From (49) and (50) follows that

$$p_0p'_0 - 2p_1p'_1 - 2p_2p'_2 - 2p_3p'_3 = \kappa^2 \quad (52)$$

must be invariant. Therefore,

$$\begin{aligned} p_0^2 + p_0'^2 - p_1^2 - p_1'^2 - p_2^2 - p_2'^2 - p_3^2 - p_3'^2 \\ = M^2 - \kappa^2. \end{aligned} \quad (53)$$

The symmetry group of this quadratic form is  $\text{SO}(6,2)$ . Relation (52) reduces the number of independent parameters from 6 to 5 and thereby  $\text{SO}(6,2)$  to  $\text{SO}(5,2)$ .  $\text{SO}(5,2)$  acts transitively on this 5-dimensional parameter space. Each point in this parameter space corresponds to a state in the two-particle state space  $\mathcal{H}_M$ . Therefore, the volume of the parameter space delivers a measure for the number of states that can contribute to the interaction term.

Given a point  $Q$  in this parameter space, then other points can be reached by applying a linear transformation of  $\text{SO}(5,2)$  to  $Q$ . There are certain transformations that do *not* change the point  $Q$ . These transformations form the subgroup  $\text{S}(\text{O}(5) \times \text{O}(2))$ . This is the isotropy subgroup or stabilizer of  $Q$ . Therefore, to obtain the multiplicity of states, we have to start from the coset space  $D_5 = \text{SO}(5,2)/\text{S}(\text{O}(5) \times \text{O}(2))$  rather than from  $\text{SO}(5,2)$ .

$D_5$  is a symmetric space. By construction  $D_5$  is isomorphic to  $\mathcal{H}_M$ . It is known from the work of Hua and Lu [14] that  $D_5$  can be represented by matrices; that is, this symmetric space is isomorphic to the real hyperball

$$\mathcal{R}_R(5, 2) = \{X \in R^{5 \times 2} \mid I - XX' > 0\}. \quad (54)$$

(See [15] for a modern introduction to symmetric spaces.) Hua [16] has calculated volumes of  $\mathcal{R}_R(5, 2)$  and other domains. In contrast to  $\mathcal{R}_R(5, 2)$   $D_5$  has an infinite volume.

Consider now a two-particle state  $|\mathbf{P}\rangle$  with 4-momentum  $P$ . There is another volume associated with  $D_5$ . This is the subspace of all points that correspond to a situation where for one of the particles  $p_0 = m$  and the other particle has reached its maximum value of  $p'_0 = P_0 - m$ . For reasons of symmetry this volume is spherical symmetric and isomorphic to the border sphere  $C_5$  of  $D_5$ .  $C_5$  has 4 dimensions. Then all states with given state  $P_0$  are confined to a volume  $\bar{D}_5$  inside of  $C_5$  and including  $C_5$ . The subspace  $\bar{D}_5$  of  $D_5$  is finite and can be mapped onto  $\mathcal{R}_R(5, 2)$  by an isometric mapping.

If  $Q = (q_1, \dots, q_5)$  is a point of  $\bar{D}_5$  that is mapped into a point  $S = (s_1, \dots, s_5)$  of  $\mathcal{R}_R(5, 2)$ , then we can establish a one-to-one relationship such that

$$q_i = r s_i, \quad (55)$$

where  $r$  is a properly chosen scaling factor. This gives us the choice to use either  $q_i$  in  $\bar{D}_5$  or  $s_i$  in  $\mathcal{R}_R(5, 2)$  as integration parameters.

To be consistent with Smith's terminology we will calculate all volumes in  $\mathcal{R}_R(5, 2)$ . We will use the notation  $V(D_5)$  for the volume that corresponds to  $\bar{D}_5$  but is calculated in  $\mathcal{R}_R(5, 2)$  and will remember that we have to apply the correct number of scaling factors  $r$ .

$C_5$  has another important property: If particle 1 is initially at rest and a second particle with a given momentum  $p'$  is added to form a two-particle state, then this state corresponds to a point on  $C_5$  as described before. Other states can be generated from this 'initial' state by the exchange of momentum. Therefore, to determine all states that are eventually involved we have first to collate all initial states.

This means, we have to perform an integration over  $C_5$  with a volume element  $d^4s/V(C_5)$ . This delivers a first volume factor of  $1/V(C_5)$ .

To collect all possible momentum changes of particle 1 we have to integrate over  $\bar{D}_5$ . From condition (37) it follows that for a given  $|\mathbf{P}\rangle$  only momentum exchanges in the subspace perpendicular to  $P$  have to be considered. Since the direction of the total momentum is undetermined (when we are constructing the interaction operator), we have to keep the integration over  $D_5$ . We compensate for this by a volume factor of  $1/V(S_4)$  where  $S_4 = SO(5, 2)/SO(4, 2) = SO(5)/SO(4)$  is the unit sphere in 4 dimensions. This reduces the number of independent parameters to 4. Let  $(s_1, \dots, s_4)$  be a new set of independent parameters corresponding to a new set  $(q_1, \dots, q_4)$ .

If we integrate over  $\bar{D}_5$  using this new parameter set, each  $s_i$  will be responsible for a contribution of  $V(D_5)^{1/4}$  to the volume of  $\bar{D}_5$ . Three of these parameters can now be mapped onto the transferred momentum  $\mathbf{k}$ . The fourth parameter  $s_4$ , obviously, corresponds to a momentum transfer within each of the particle momentum, without any momentum transfer between the particles. Such transitions contribute to the volume of  $C_5$ . We can perform the integration over  $s_4$  and obtain a correcting factor to the already calculated volume  $V(C_5)$  of  $V(D_5)^{1/4}$ .

There are three more factors that contribute to the multiplicity of momentum states. One is related to the spin components of the particle states, which give each momentum state a multiplicity of  $2\pi$  because of the periodicity of spin states. The other factor is related to the (relative) phases of the momentum states within multiparticle states. By adding another factor of  $2\pi$  we take into account this degree-of-freedom. Finally, remember that there are two terms  $\gamma^\mu p'_\mu$  and  $\gamma'^\mu p_\mu$  that contribute to the interaction. This delivers a factor of 2.

After extracting these constant factors from the integral we are left with an integration over the  $p'$  parameter space where now the integrand should enter with a multiplicity of one within the  $p'$ -parameter space - provided that we have correctly captured all factors that determine any multiplicities. Collecting these factors we end up with

$$8\pi^2 V(D_5)^{1/4} / (V(S_4) V(C_5)). \quad (56)$$

This is essentially Wyler's formula.

The volumes  $V(D_5)$  and  $V(C_5)$  have been calculated by L. K. Hua [16].  $V(S_4)$  is the volume of the unit sphere  $S_4$  in 4 dimensions. With

$$V(C_5) = \frac{8\pi^3}{3}, \quad (57)$$

$$V(D_5) = \frac{\pi^5}{2^4 5!}, \quad (58)$$

$$V(S_4) = \frac{8\pi^2}{3} \quad (59)$$

we obtain

$$\frac{9}{8\pi^3} \left( \frac{\pi^5}{2^4 5!} \right)^{1/4}. \quad (60)$$

If we identify this value with the coupling constant  $e^2/(2\pi)^2 = \alpha/\pi$  in the S-matrix element for Møller scattering we obtain a value for  $\alpha$

$$\alpha = \frac{9}{8\pi^4} \left( \frac{\pi^5}{2^4 5!} \right)^{1/4} = 1/137.03608245. \quad (61)$$

Although intended only as an estimate, this result is in agreement with experimental values in five-parts in ten-million. (A value of 137.035 999 93(52) has been determined from the magnetic moment of the electron [19].)

We can easily convince ourselves that the scaling factors  $r$  either cancel or are absorbed in the volume element  $d^3q$ .

This result means more than just an estimate of the coupling constant. It delivers numerical support of our statement that only states of the same irreducible representation of the Poincaré group contribute to the interaction term. In other words, it can be considered as a confirmation of our implementation of the projection operator into the interaction term.

In this section we have obtained a theoretical justification for the scheme presented in the article by F. D. Smith, Jr. [18]. Concerning coupling constants of other interactions, the reader is referred to Smith's article.

## XV. QUANTUM GRAVITY

Let us now examine the second corrective term (21) that has been identified in the beginning. Consider the representation of the generators of the  $SO(3,2)$  in the following form

$$l_{ab} = x_b p_a - x_a p_b, \quad (62)$$

where the ‘momentum’ operators  $p_a$  are represented by differential operators  $-i\partial/\partial x^a$ . In writing down this expression we have embedded the tangential space-time into a 5-dimensional pseudo-Euclidean space by adding  $x_4$  as an additional (time-like) coordinate.

Within a neighborhood  $\mathcal{N}$  of the point  $x_\mu = 0, x_4 = 1$ , group contraction approximates the operators

$$l_{\mu 4} = x_4 p_\mu - x_\mu p_4, \quad \mu = 0, \dots, 3, \quad (63)$$

by momentum operators  $p_\mu$ .

A better approximation is obtained if the second term on the right of (63)

$$c_\mu = -x_\mu p_4 \quad (64)$$

is added. This term delivers a contribution to the difference between translations and exact  $SO(3,2)$  operators that is of the first order in  $x$  within the neighborhood  $\mathcal{N}$ . This operator has the structure

of a ‘translation’ operator in the direction of  $x_4$  weighted with  $x_\mu$ . So we can say that this corrective term leads out of the tangential space-time into the direction of  $x_4$  that is perpendicular to tangential space-time. Or, in other words, it adds curvature to the originally flat space-time.

Let us see how this generation of curvature depends on the properties of a two-particle state. As we have proceeded before, we start from a ‘constant-of-motion’ given by the constant operator

$$L^{ab}L_{ab} = (l^{ab} + l'^{ab})(l_{ab} + l'_{ab}). \quad (65)$$

Again we assume that within the neighborhood  $\mathcal{N}$  the relations (11) is valid. If we evaluate the product (65) with  $l_{\mu 4}$  approximated by

$$t_\mu := p_\mu + c_\mu \quad (66)$$

and take only terms that are quadratic and linear in  $p$ , then within the linear terms we obtain mixed terms of operators of particles 1 and 2 in the form of  $p_\mu c'^\mu$  and  $p'_\mu c^\mu$ . These terms define a correction to those terms that we would have obtained also in a pure Poincaré invariant situation.

If we evaluate the mixed terms for a two particle state with well-defined total energy-momentum  $P$  we obtain contributions in the form

$$\langle \phi(\mathbf{p} + \mathbf{k}) | p_\mu | \phi(\mathbf{p}) \rangle \langle \phi(\mathbf{p}' - \mathbf{k}') | c'^\mu | \phi(\mathbf{p}') \rangle + \langle \phi(\mathbf{p} + \mathbf{k}) | c^\mu | \phi(\mathbf{p}) \rangle \langle \phi(\mathbf{p}' - \mathbf{k}') | p'_\mu | \phi(\mathbf{p}') \rangle. \quad (67)$$

Since in the total momentum must be constant, we have  $\mathbf{k} = \mathbf{k}'$ .

From this we can directly read that the curvature generating correction, to the kinematics of particle 2 caused by  $\langle \phi(\mathbf{p}' - \mathbf{k}) | c'^\mu | \phi(\mathbf{p}') \rangle$ , is weighted by the 4-momentum amplitude  $\langle \phi(\mathbf{p} + \mathbf{k}) | p^\mu | \phi(\mathbf{p}) \rangle$  of particle 1 and vice versa. In more classical terms: the curving of tangential space-time is proportional to the distribution of matter, if we consider particle 2 as a test-particle and particle 1 as a representative of ‘the rest of the world’.

This result clearly tells us that there is no uniform curvature, unless matter is uniformly distributed as in the classical model of de Sitter space-time.

We will now formulate the interaction term  $p_\mu c'^\mu$  as an operator in Fock space, with  $\Phi^\dagger$  and  $\Phi$  as emission and absorption operators, in the same way as in the case of QED. (A similar procedure is valid for the other term  $p'_\mu c^\mu$ .) Consider the contribution

$$\Phi^\dagger(\mathbf{p} + \mathbf{k}) p_\mu \Phi(\mathbf{p}) \Phi^\dagger(\mathbf{p}' - \mathbf{k}) c'^\mu \Phi(\mathbf{p}'). \quad (68)$$

By collecting all terms that contribute to given  $\mathbf{p}$  and  $\mathbf{k}$  we obtain



$$\Phi^\dagger(\mathbf{p} + \mathbf{k}) p_\mu \Phi(\mathbf{p}) \Gamma^\mu(\mathbf{k}) \quad (69)$$

with

$$\Gamma^\mu(\mathbf{k}) := \int dV(p') \Phi^\dagger(\mathbf{p}' - \mathbf{k}) c^\mu \Phi(\mathbf{p}'). \quad (70)$$

If we replace  $\Gamma^\mu(\mathbf{k})$  by a potential  $G^\mu(\mathbf{k})$ , in the same way as we have introduced  $A^\mu(\mathbf{k})$ , we obtain from (69) - at least formally - an interaction term in space-time

$$\Phi^\dagger(x) \partial_\mu \Phi(x) \times G^\mu(x). \quad (71)$$

This operator can - in principle - be used as an interaction term within a properly defined perturbation algorithm in a 5-dimensional pseudo-Euclidean space.

Within our picture of ‘test-particle’ and ‘massive body’ we come to the following interpretation of this interaction term.  $G^\mu(x)$  describes a potential that acts on the test-particle in that it adds to its momentum in a way that curves space-time. The magnitude of this potential is controlled by the distribution of 4-momentum of the massive body as a multiplicative factor. Of course, the test-particle also acts on the massive body by the other term  $p'_\mu c^\mu$ . Therefore, this statement can be generalized to: *curving of space-time is proportional to the distribution of energy-momentum*.

If there were a well-established theory of quantum gravity, we would now proceed and try to bring this interaction term into a form that can be compared with the established theory - as we have done in the case of QED. But at present, we only have at our disposal a theory of gravitation in the classical domain. Therefore, we will find out what the structure of this interaction term can tell us about the properties of a classical limit to this quantum mechanical system. This will give us a hint whether or not our model is suited to describe a realistic gravitation-like interaction.

## XVI. CLASSICAL CORRESPONDENCE LIMIT

We already have found that the interaction term causes a curving of space-time, and that this curving is not uniform as one might expect if one has the cosmological model of ‘de Sitter space-time’ in mind. Instead it is proportional to the distribution of 4-momentum. If we start from a flat space-time then the interaction will cause modifications of the metric tensor. The central question is: what are the parameters, in the classical limit, that these modifications depend on?

Obviously, the curving does not depend on any internal property of the test-particle but is directly linked as an additive term to its momentum, thereby revealing an ‘universal’ character of the curving mechanism. The property of universality gives us a good chance that we can express the bending of

space-time that our test-particle experiences by a modification of the metric tensor that is equally valid for any other test-particle. This then leads us to the concept of a non-Euclidean space-time with its curvature determined by the distribution of matter.

Our concept is invariant with respect to the symmetry operations of  $SO(3,2)$ . Therefore, it must be possible to find a formulation of a classical limit that is covariant with respect to the operations of  $SO(3,2)$ . We know that with respect to the neighborhood of any given point these operations can be understood as the application of the homogeneous Lorentz group (as a subgroup of  $SO(3,2)$ ) and, in addition, of four operations that can be approximated by translations under certain conditions, but, in general, include a bending of space-time. From this we conclude, firstly, that the relation between curvature and distribution of matter must be covariant with respect to the homogeneous Lorentz group. And, secondly, that covariance has to be extended to a non-Euclidean metric, which means general covariance in the sense of classical general relativity.

We can understand the interaction term (71) as a description of the variation of the metric tensor, due to the interaction. Then the form of (71) tells us that the variation at the point  $x$  is related to the amount of matter *at the same point  $x$* . This means that the relation between curvature and matter may contain the metric tensor itself and also its derivatives at the point  $x$ .

As long as we restrict ourselves to ‘small’ curvatures, it will be sufficient to consider only such terms that contain no higher than the second derivative of the metric tensor and that are linear in the second differential quotient. As we know from general relativity [20], under these conditions there is only one tensor with vanishing divergence that can be built from the metric tensor, namely

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R. \quad (72)$$

(The definitions of  $R_{\mu\nu}$  and  $R$  can be found in [20] or any textbook on general relativity.)

On the other hand, a covariant description of the distribution of 4-momentum is uniquely given by the energy-momentum tensor  $T_{\mu\nu}$ . Its divergence must vanish because of energy-momentum conservation.

Then the properties of the interaction term, as discussed above, determine that the curvature-generating mechanism must be proportional to the energy-momentum tensor. This leads us to the well-known field equations of general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu} \quad (73)$$

with an unknown ‘coupling constant’  $\kappa$ .

## XVII. CURRENT-CURRENT COUPLING

In deriving expression (24) we had collected all contributions of magnitude  $p^2$  and  $p^1$ . We had neglected contributions of the form  $\gamma^\mu \gamma'_\mu$  because they are of magnitude  $p^0$  and, therefore, expected to be very weak. Such terms are of the same order of  $p$  as  $j^{\mu\nu} j_{\mu\nu}$ , which under transformations of the Poincaré group behaves like a contribution to a mass term. This prompts the question whether under these circumstances we can still consider the latter as constant. We do not know the answer yet.

In any case, there may be situations where we are able to observe contributions of  $\gamma^\mu \gamma'_\mu$ . So let us take up this term now and consider its contribution

$$\bar{b}(\mathbf{p}') \gamma_\mu b(\mathbf{p}'') \bar{b}(\mathbf{p}''') \gamma^\mu b(\mathbf{p}''') \quad (74)$$

to the corresponding interaction term in Fock space.

Remembering how we have derived QED, we will replace a pair of emission and absorption operators by operators of a bookkeeping system or potentials, respectively, in order to linearize the interaction term. Let us rename the operators of the first particle by  $e$  and of the second by  $\mu$  and then replace two operators in (74) by potential operators with the names  $\nu_e$  and  $\nu_\mu$ . With this we modify (74) to

$$\bar{e}(\mathbf{p}') \gamma_\mu \nu_e(\mathbf{p}'') \bar{\nu}_\mu(\mathbf{p}''') \gamma^\mu \mu(\mathbf{p}'''). \quad (75)$$

This means, we have split the interaction term into two parts that are connected by potentials as causal links. Notice that the analogue to the integral over  $p'$  in (43) is now the sum over the four spinor components in (75).

We can specialize (75) to a situation where the second part of the interaction term is evaluated at a large distance from the first part. Then, as we have learned from QED, the linking quanta of the potential become mass-less quanta taking care of the exchange of spin (and momentum) in this case. For mass-less spinors the four spin components decouple into two-component states of left- and right-handed helicity. We identify these states with particles of positive energy, running forward in time, and anti-particles of negative energy, running backward in time, respectively. These can be separated by projection operators (compare (9) for  $\gamma_5$  in Weyl representation)

$$\frac{1}{2}(1 - \gamma_5) \quad \text{and} \quad \frac{1}{2}(1 + \gamma_5) \quad (76)$$

from the four-component states. The potentials in (75) are linked, by construction, to particles. Therefore, we insert a projection to ‘particles’, which have a left-handed helicity, into (75). We then

obtain a basic building block in the well-known form of weak interaction (we have dropped a trivial factor of  $1/4$ )

$$\bar{e}(\mathbf{p}') \gamma_\mu (1 + \gamma_5) \nu_e(\mathbf{p}'') \bar{\nu}_\mu(\mathbf{p}''') \gamma^\mu (1 + \gamma_5) \mu(\mathbf{p}'''). \quad (77)$$

(We have used the commutation relations of the  $\gamma$ -matrices to bring the projection operator of the myon-term to the same position as in the electron-term.) This contribution describes the conversion of a myon and an electron-neutrino to an electron and a myon-neutrino. It implies a ‘maximum parity violation’ of the interaction provided by this term.

Let us now replace the electron-neutrino in (77) by an anti-electron-neutrino, which according to our understanding has a right-handed helicity and negative energy. In the standard treatment of fermion fields, states with negative energy running backward in time are handled as anti-particles with positive energy running forward in time. This reinterpretation implies a reflection with respect to time. The operation of time reflection applied to a Dirac spinor is given by

$$T : x \longrightarrow x' = (-x_0, \mathbf{x}), \quad \psi(x) \longrightarrow \psi'(x') = \pm \gamma_0 \psi(x) \quad (78)$$

(see e.g. [11]). Since  $\gamma_5$  anticommutes with  $\gamma_0$ , time reflection, therefore, means a replacement of right- by left-handed helicity. So again only left-handed states are found in this contribution if we relate it to the standard formulation of weak interaction. This contribution describes the decay of a myon to an electron, an anti-electron-neutrino and a myon-neutrino.

Although we do not know yet what causes the difference between  $e$  and  $\mu$ , we can identify a difference between  $\nu_e$  and  $\nu_\mu$  in the following respect. Since  $\nu_e$  and  $\nu_\mu$  represent far distant  $e$  and  $\mu$ , respectively, they have *different* properties in so far as  $\nu_e$  links only to  $e$  and  $\nu_\mu$  only to  $\mu$ . This reflects the empirical difference of electron- and myon-neutrinos.

We had good reasons to restrict our consideration to massless quanta. With a neutrino we connect the idea of a *free* particle, which means a quantum that is exchanged over a large distance. As such it must have a zero mass. Also, because of the weakness of the interaction, there is hardly a chance to observe a neutrino that is absorbed immediately after its emission.

## XVIII. COMPOUND STATES

So far we have studied only processes where two single-particle states within  $\mathcal{H}_\mathcal{O}$  combine to form a two-particle state and eventually disintegrate again.

If we consider compound states there is no reason to expect that their constituent single-particle states are always locatable in tangential space-time, if only the resulting compound state can be located within  $\mathcal{H}_O$ .

How can we describe states that do not belong to  $\mathcal{H}_O$  from the view and in the language of an observer in tangential space-time?

Consider the operator  $l_{04}$ , the SO(3,2) counterpart of the energy or time-translation operator  $p_0$ . It generates a rotation in the SO(3,2) Hilbert space  $\mathcal{H}$  within the 0-4-plane. A rotation by  $\pi/2$  transforms  $l_{i4}$  into  $l_{i0}$ ,  $i = 1, 2, 3$ , but leaves  $l_{04}$  invariant. Therefore,  $l_{i0}$  together with  $l_{40}$  can generate a kind of shadow tangential space-time with Hilbert space  $\widetilde{\mathcal{H}}_O$  in the same way as  $l_{i0}$  generate the normal one.  $l_{ik}$  still generate space-like rotations but the meaning of momentum and boost operators are interchanged. States of  $\widetilde{\mathcal{H}}_O$  can be labeled by eigenvalues of operators  $q_i$  that are obtained by applying a contraction limit to  $l_{i0}$  using condition (11), in a form adapted to  $\widetilde{\mathcal{H}}_O$ .

The interesting point is that the energy operator of these states is the same as of states in  $\mathcal{H}_O$ . Therefore, the shadow states will contribute to the total energy and should thereby add to gravitation. On the other hand, they cannot participate in the electromagnetic interaction with  $\mathcal{H}_O$ , because their operators  $q_i$  cannot exchange ‘momentum’ with the operators  $p_i$  of  $\mathcal{H}_O$  - at least not by the mechanism that we have identified as electromagnetic interaction. It is tempting to regard these states as a kind of ‘dark matter’.

Consider now special types of states that belong neither to  $\mathcal{H}_O$  nor to  $\widetilde{\mathcal{H}}_O$  nor to any linear combination of regular and shadow states. We define such states by exchanging one or two momentum components  $p_i$  by the corresponding  $q_i$ .

This leads to the following configurations:  $(p_1, p_2, q_3)$ ,  $(p_1, q_2, p_3)$ ,  $(q_1, p_2, p_3)$  and  $(p_1, q_2, q_3)$ ,  $(q_1, q_2, p_3)$ ,  $(q_1, p_2, q_3)$ . We can easily convince ourselves that  $p_i$  commute with  $q_k$  if  $i \neq k$ , by using the commutation relations of SO(3,2) together with (a generalized form of) condition (11). This means that the operator triplets deliver as good quantum numbers as the  $p_k$  of regular states, despite the somewhat coarse way of their construction. It would be hard to understand if such states were not to be occupied in the same way as states of  $\mathcal{H}_O$  or  $\widetilde{\mathcal{H}}_O$ . Therefore, we have to conclude that such states are an inevitable and integral part of the SO(3,2) model.

These states cannot be localized in tangential space-time because one or two of the momentum components are missing. Therefore, these states do not belong to any irreducible representation of the Poincaré group, which means that they cannot appear as free particles. But since these states have one or two regular momentum components they may be able to exchange momentum with

regular particles by electromagnetic interaction. Remembering the factors that contribute to the estimate of the electromagnetic coupling constant we can, at least formally, ascribe charges of  $1/3$  and  $2/3$  of  $e$  to these states.

If we intend to construct compound states that shall appear as particles in the tangential space-time, we have to combine the individual states in such a way that either the  $q_i$  compensate or add up to a symmetric configuration, so that the compound state corresponds finally to a representation of the Poincaré group.

Similar properties has been found empirically within the quark model and have triggered the formulation of quantum chromodynamics. We, therefore, tend to label the three modifications within each of the two groups by ‘color’ quantum numbers, and identify the first group with ‘up quarks’ and the second with ‘down quarks’.

Let  $a_r^\dagger, a_r, a_b^\dagger, a_b, a_g^\dagger, a_g$  be creation and annihilation operators of quarks with colors  $r, b, g$ . Then the operators

$$\begin{aligned} T_+ &= a_r^\dagger a_b, \quad T_- = a_b^\dagger a_r \\ B_+ &= a_r^\dagger a_g, \quad B_- = a_b^\dagger a_g \\ C_+ &= a_g^\dagger a_b, \quad C_- = a_g^\dagger a_r \end{aligned} \tag{79}$$

exchange the members of a triplet. It can be shown that together with the counting operators

$$\begin{aligned} B &= a_r^\dagger a_r + a_b^\dagger a_b + a_g^\dagger a_g \\ T &= \frac{1}{2}(a_r^\dagger a_r - a_b^\dagger a_b) \\ N &= \frac{1}{3}(a_r^\dagger a_r + a_b^\dagger a_b - 2a_g^\dagger a_g) \end{aligned} \tag{80}$$

they form the Lie algebra of  $SU(3)$  [21].

Since none of the colors are preferred to the others, any interaction term that involves these states must be invariant with respect to the exchange operations of  $SU(3)$ . This means that a generalization of the interaction term of QED to compound states built from ‘quarks’, will lead to an interaction term of a similar structure as that of quantum chromodynamics.

As mentioned above, a rotation generated by  $l_{04}$  transforms  $l_{i4}$  into  $l_{i0}$ ,  $i = 1, 2, 3$ , and vice versa.  $l_{04}$  is the *exact*  $SO(3,2)$  counterpart of time-translation. Therefore, compound states that are built up from contributions of  $p_i$  and  $q_i$ , in a way that is symmetric with respect to operations of  $l_{04}$ , can be stable over an extremely long period of time. This supports the suspicion that ‘quark’ states play an essential role in the formation of compound states.

We have sketched here a possible correspondence to QCD in order to illustrate that very promising structures can be found within compound states. These deserve a deeper evaluation, but such an evaluation is beyond the scope of the present article.

## XIX. SPIN FOAMS REVISED

Our perturbative approach to the  $SO(3,2)$  spin foam model has led us to familiar structures of perturbative quantum field theory. In reverse, we should feel entitled to regard the corresponding sums over Feynman graphs as a perturbative description of our spin foam model. This gives the ‘vertices’ of the spin foam model a precise meaning, which allows accurate numerical calculations. The ‘edges’ of spin foams then have to be identified as incoming and outgoing lines of Feynman graphs and are, therefore, labeled by spin-1/2 representations of the Poincaré group instead of  $SO(3,2)$ . Internal lines of Feynman graphs corresponding to contraction functions have to be considered as part of the perturbation algorithm and should not be related to edges of spin foams.

A similar description of spin foams by Feynman graphs has been proposed by M. Reisenberger and C. Rovelli [22].

## XX. QUANTUM COSMOLOGY

We have found strong evidence for a fundamental symmetry with the structure of the group  $SO(3,2)$ , with the homogeneous Lorentz group as a subgroup and the full Poincaré group serving as an approximate symmetry. The QED part of our model verifies the  $SO(3,2)$  symmetry from subatomic to macroscopic scales, whereas the gravitational part has the potential to verify the model at least up to the borders of our solar system, and possibly up to cosmologic scales.

The model consistently forms the structure of space-time by the gravitational part and delivers a means to probe space-time by long range photons. This should make it an useful quantum mechanically based cosmologic model reaching from subatomic to cosmologic scales.

In studying compound systems we have noticed that there is another tangential space orthogonal to the first. Both spaces are connected by the energy operator, and time *rotates* rather than *shifts* one space into the other. This brings up an interesting point: By randomly selecting a tangential space-time the particle world is divided, in principle, into three kinds of matter:

a) states that are located in the tangential space-time (matter described by presentations of the Poincaré group acting in  $\mathcal{H}_O$ ),

- b) states that are located in the orthogonal shadow space (dark matter),
- c) states that are defined by mixed quantum numbers of both spaces (quarks).

This defines a mechanism of symmetry breaking caused by using tangential space-time as the basis of the theory.

Of course, there are states that belong to tangential spaces between those of case a and b. Concerning these states, the electromagnetic part probably has to be generalized for the exchange of photons over very long distances back in time. There the rotation from  $\mathcal{H}_O$  towards  $\widetilde{\mathcal{H}}_O$  can become noticeable, and the model will clearly guide us how to accomplish this generalization.

There has been some concern that the  $SO(2)$  subgroup of rotations in the 0-4-plane could lead to unphysical time loops (Grandfather paradox). In fact, the operator  $l_{04}$  corresponding to time translations generates rotations of the Hilbert space  $\mathcal{H}$ . However, no experiment would be able to observe a rotation of the *entire*  $\mathcal{H}$ . Therefore, the  $l_{04}$  formally generates a kind of periodic background time, but to an observer it appears like a translation with an infinite range. An appropriate picture of the structure of this background time is that of a helix rather than a circle.

Nevertheless, there are means to observe effects of a rotation if we probe space-time by observing photons from events at very large distances back in the past, as mentioned before. This may show the observed red shift of distant galaxies in a new light.

Background time is useful for embedding events that are defined as transitions between quantum states. But it is the sequence of events that forms our imagination of time and that finally defines the physical properties of time. There is definitely no periodicity connected with a long sequence of statistical events. Especially, within our model the grandchild will never have a chance to meet his grandfather in a time loop.

## XXI. CONCLUSION

We have established a model of space-time and of particle physics therein that in some sense is parallel to the standard model in that it describes interactions from a different point of view. However, this model has proven much more stringent in that it

- delivers models of all four known types of interactions (and only these) by the same symmetry principle,
- allows to determine coupling constants,
- delivers a particle spectrum consisting of massive leptons (so far we do not know what makes up the difference between electron, myon and tau), photons, neutrinos, quarks and compound states



made up by quarks,

- explains gauge invariance (in case of QED) and SU(3) invariance (in case of QCD) and violation of parity (in case of weak interaction),
- describes the formation of space-time as a combined action of electromagnetic and gravitational interaction.

Space-time has formally been obtained as tangential space-time by contracting the SO(3,2) symmetry group to the Poincaré group. This does not come as a surprise since this is a consequence of the representations that we have chosen. However, in connection with the properties of the electromagnetic interaction term, which is ‘local’ in space *and time*, it adopts qualities of an observable physical space-time continuum: the interaction term allows - in principle - to perform measurements of particles at ‘points’ in space and time. This realized the notion of *events in space and time*.

Also the auxiliary ‘photon’ field obtains ‘real’ physical properties identical to the empirical electromagnetic field in the sense that the interaction term allows us to probe its action on ‘charged particles’ at different points of space-time. In addition, photons represent *causal links* connecting events that are caused by the interaction term.

All results described in this article have been obtained by replacing the Poincaré symmetry of the standard model by a de Sitter symmetry. Except for the presumption of large quantum numbers no additional assumptions like gauge invariance or ‘higher’ symmetries have been made.

The implications of this replacement for particle physics makes it very likely that the de Sitter group SO(3,2) defines a fundamental symmetry of nature, well-hidden by innocent-looking interaction terms. Only to the extent that interactions can be neglected it can be approximated by the Poincaré group.

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